1691 Algebraic Inequalities

Problem Solving

Old and New Problems for Mathematical Olympiads

Panagiote Ligouras

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The book starts with a collection by Aassila M., Andreescu T., Barbeau E. J., Beckenbach E., Bourgade P., Bulajich Manfrino R., Cirtoaje V., Dorrei H., Dospinescu G., Enescu B., Engel A., Feng Z., Gomez Ortega J.A., Hardy G.H., Klamkin M., Kolev E., Lascu M., Larson L., Littlewood J.E., Mitrinovic D., Mushkarov O., Negut A., Nikolov N., Panaitopol L., Pham Kim Hung, Pòlya G., Rabinowitz , reiman I., Shawyer B.L.R., Stergiou B., Soulami T., Todev R., Xiong Bin and of many other friends. Its intention is to bring together, in the successive editions, all significant algebraic inequalities. This will also be done by finding out the author, year in which it was created and grade of difficulty on a scale of 1 to 5 (banal, easy, medium, requiring effort and difficult).

We haven't classified the problems according to the techniques used to solve them, or by their difficulty, in order to avoid limiting the creativity and freedom of the person solving them.

In writing this book, the website www.mathlinks.ro has been very useful. It is created and maintained by Dr Valentin Vornicu, who is totally dedicated to all the kinds of mathematics that help in preparing for national mathematics competitions and the International Mathematics Olympics.

On the website's forum, these mathematical inequalities can be discussed much further. Alternative solutions can be suggested, along with new problems that may help to develop the knowledge of friends visiting the website. This also provides an opportunity to underline the importance of mathematics as a discipline, for the development of men and also of society. The problems that are marked t=329751 or similar are present on the forum of the website www.mathlinks.ro.

Part I

------------------------------ **AI1 - 1** ------------------------------

Let a be real number ($a \in R$). Prove that:

$$a)\ a+\frac{1}{a} \geq 2, \quad a>0 \qquad\qquad b)\ a+\frac{1}{a} \leq -2, \quad a<0$$

------------------------------ **AI1 - 2** ------------------------------

Let a be non-negative real number ($a \geq 0$). Prove that:

$$a^5 - a^2 + 3 \geq a^3 + 2$$

Titu Andreescu, 2004

------------------------------ **AI1 - 3** ------------------------------

Let a be non-negative real number ($a \geq 0$). Prove that:

$$5\left(a^2 - a + 1\right)^2 \geq 2\left(1 + a^4\right)$$

Panagiote Ligouras, 2009

------------------------------ **AI1 - 3** ------------------------------

Let a be non-negative real number ($a \geq 0$). Prove that:

$$2\left(a^2+1\right)^3 \geq \left(a^3+1\right)\left(a+1\right)^3$$

------------------------------ **AI1 - 4** ------------------------------

Let a be non-negative real number ($a \geq 0$). Prove that:

$$4\left(a^3+1\right)^3 \geq \left(a^4+1\right)\left(a^2+1\right)\left(a+1\right)^3, \quad a \geq 0$$

Panagiote Ligouras, 2009

------------------------------ **AI1 - 5** ------------------------------

Let a be non-negative real number ($a \geq 0$). Prove that:

$$2\left(a^3+1\right)^4 \geq \left(a^4+1\right)\left(a^2+1\right)^4$$

$$\left(1+\frac{1}{\sqrt{a}}\right)\left(1+\frac{1}{\sqrt{8-a}}\right)\geq\frac{9}{4},\quad 8>a>0$$

Gazeta Matematica, 2002

------------------------------ **AI1 - 12** ------------------------------

Let a be real number ($a\in R$) such that $-1<a<1$. Prove that:

$$\sqrt[4]{1-a^2}+\sqrt[4]{1-a}+\sqrt[4]{1+a}\leq 3$$

The Mathscope, n.216.3

------------------------------ **AI1 - 13** ------------------------------

Let a be positive real number ($a>0$). Prove that:

$$\frac{a^3+1}{a^2+1}\geq\sqrt{a^2-a+1}\geq\sqrt[4]{\frac{a^4+1}{2}}$$

CRUX, 35(4), p. 254, 2009

------------------------------ **AI1 - 14** ------------------------------

Let a be non-negative real number ($a\geq 0$). Prove that:

$$\sqrt{a}+\sqrt[3]{a}+\sqrt[6]{a}\leq a+2$$

The Mathscope, n.332.5

------------------------------ **AI1 - 15** ------------------------------

Let a be non-negative real number ($a\geq 0$). Prove that:

$$\sqrt{a}+\sqrt[3]{a^2}+\sqrt[6]{a^5}\leq 2a+1$$

Panagiote Ligouras, Mathlinks, 2009

------------------------------ **AI1 - 16** ------------------------------

Let a be real number such that $a\geq -2$. Prove that:

$$\sqrt{a+2}+\sqrt[3]{a+2}+\sqrt[6]{a+2}\leq a+4$$

Panagiote Ligouras, Mathlinks, 2009

------------------------------ **AI1 - 17** ------------------------------

Problems with 2 variables

--------------------------------- **AI2 - 21** ---------------------------------

Let a, b be non-negative real numbers ($a,b \ge 0$). Prove that:

$$a+b \ge 2\sqrt{ab}$$

--------------------------------- **AI2 - 22** ---------------------------------

Let a, b be real numbers ($a,b \in R$). Prove that:

$$a^2+b^2 \ge 2ab$$

--------------------------------- **AI2 - 23** ---------------------------------

Let $a,b \ge 0$. Prove that:

$$a^3+b^3 \ge ab\left(a+b\right)$$

--------------------------------- **AI2 - 24** ---------------------------------

Let $a,b \ge 0$. Prove that:

$$a^4+b^4 \ge ab\left(a^2+b^2\right)$$

--------------------------------- **AI2 - 25** ---------------------------------

Let $a,b \ge 0$. Prove that:

$$a^5+b^5 \ge ab\left(a^3+b^3\right) \ge a^2b^2\left(a+b\right)$$

--------------------------------- **AI2 - 26** ---------------------------------

Let $a,b \in R$. Prove that:

$$\left(a+b\right)^2 \ge 4ab$$

AI2 - 33

Let $a,b>0$. Prove that:

$$\frac{a^2+b^2}{a+b}\geq\frac{a+b}{2}$$

AI2 - 34

Let $a,b>0$. Prove that:

$$\frac{a+b}{a^2+b^2}\leq\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$$

AI2 - 35

Let $a,b>0$. Prove that:

$$\frac{ab}{a+b}\leq\frac{a+b}{4}$$

AI2 - 36

Let $a,b>0$. Prove that:

$$\frac{4}{a+b}\leq\frac{1}{a}+\frac{1}{b}$$

AI2 - 37

Let $a,b>0$ such that $a+b=1$. Prove that:

$$\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2\geq\frac{25}{2}$$

AI2 - 38

Let $a,b\geq 0$ and $n\in N^*$. Prove that:

$$a+b\geq\sqrt[n]{a^{n-1}b}+\sqrt[n]{ab^{n-1}}$$

------------------------------ **AI2 - 45** ------------------------------

Let $a,b \in R^*$. Prove that:

$$\frac{2a^2+3b^2}{2a^3+3b^3}+\frac{2b^2+3a^2}{2b^3+3a^3} \le \frac{4}{a+b}$$

The Mathscope, n 285.2

------------------------------ **AI2 - 46** ------------------------------

Let $a,b>0$. Prove that:

$$\frac{1}{ab} \ge \frac{a}{a^4+b^2}+\frac{b}{a^2+b^4}$$

Russia, 1995

------------------------------ **AI2 - 47** ------------------------------

Let $a,b>0$ and $k \ge 1$. Prove that:

$$\frac{a+kb}{(b+ka)^2}+\frac{b+ka}{(a+kb)^2} \ge \frac{a+b}{a^2+(k-1)ab+b^2}$$

MathLinks, t=255035, 2009

------------------------------ **AI2 - 48** ------------------------------

Let $a,b>0,\ \ a+b=1$. Prove that:

$$\frac{1}{1-\sqrt{a}}+\frac{1}{1-\sqrt{b}} \ge \frac{2\sqrt{2}}{\sqrt{2}-1},\ \ a+b=1,\ \ a,b>0$$

------------------------------ **AI2 - 49** ------------------------------

Let $a,b>0,\ \ a+b=2$. Prove that:

$$a\sqrt{\frac{b}{a^2+1}}+b\sqrt{\frac{a}{b^2+1}} \le \sqrt{2}$$

------------------------------ **AI2 - 50** ------------------------------

Let $a,b>0,\ m \in N,\ m \ge 2,\ r \in R,\ rm \ge 1,\ ab=r^2$. Prove that:

$$\frac{1}{(1+a)^m}+\frac{1}{(1+b)^m} \ge \frac{2}{(1+r)^m}$$

Arkady Alt, CRUX, 34(2), n. 3319, 2008

------------------------------- **AI2 - 57** -------------------------------

Let $a,b>0$. Prove that:

$$a^3+b^3\geq a+b+2ab-2$$

Norman Schaumberger, N.Y.S.M.T.J., n.153, 1984

------------------------------- **AI2 - 58** -------------------------------

Let $a,b>0$. Prove that:

$$a^b+b^a>1$$

Kàlmàn Szabò, T.M.G., n. 67.H, 1983

------------------------------- **AI2 - 59** -------------------------------

Let $a,b\in R$ such that $|a|,|b|<1$. Prove that:

$$|ab+1|>|a+b|$$

PARABOLA, Q591, 1984

------------------------------- **AI2 - 60** -------------------------------

Let $a,b\geq 0$. Prove that:

$$\frac{1}{(1+a)^2}+\frac{1}{(1+b)^2}\geq\frac{1}{1+ab}$$

------------------------------- **AI2 - 61** -------------------------------

Let $0\leq x,y\leq 1$. Prove that:

$$\sqrt{1+x^2}+\sqrt{1+y^2}+\sqrt{(1-x)^2+(1-y)^2}\geq\left(1+\sqrt{5}\right)(1-xy)$$

------------------------------- **AI2 - 62** -------------------------------

Let $a,b>0$. Prove that:

$$\sqrt[3]{2(a+b)\left(\frac{1}{a}+\frac{1}{b}\right)}\geq\sqrt[3]{\frac{a}{b}}+\sqrt[3]{\frac{b}{a}},\quad a,b>0$$

Czech and Slovakia, 2000

AI2 - 69

Let $a+b\le 1,\ \ a,b>0$. Prove that:

$$\frac{1}{a^3+b^3}+\frac{1}{a^2b}+\frac{1}{ab^2}\ge 20$$

Cao Minh Quang

AI2 - 70

Let $ab=1,\ \ a,b>0$. Prove that:

$$\frac{a^3}{1+b}+\frac{b^3}{1+a}\ge 1$$

Le Thanh Hai

AI2 - 71

Let $a,b\ge 1$. Prove that:

$$3\left(\frac{a^2-b^2}{8}\right)+\frac{ab}{a+b}\ge\sqrt{\frac{a^2+b^2}{8}}$$

Yugoslavia, 1991

AI2 - 72

Let $x,y\in\left[1,\frac{3}{2}\right]$. Prove that:

$$x^2+y^2\ge x\sqrt{3-2y}+y\sqrt{3-2x}$$

Moldova, 2001

AI2 - 73

Let n be a positive integer, and let x and y be a positive real numbers such that $x^n+y^n=1$. Prove that

$$\frac{1}{(1-x)(1-y)}>\left(\sum_{k=1}^{n}\frac{1+x^{2k}}{1+x^{4k}}\right)\cdot\left(\sum_{k=1}^{n}\frac{1+y^{2k}}{1+y^{4k}}\right)$$

IMO Shortlist 2007

Problems with 3 variables

--- **AI3 - 79** ---

Let a, b, and c be positive real numbers such that $a^2+b^2+c^2=1$. Prove that:

$$\frac{a}{1-a^2}+\frac{b}{1-b^2}+\frac{c}{1-c^2}\geq\frac{3\sqrt{3}}{2}$$

Sefket Arslanagic, CRUX, n. 2738

--- **AI3 - 80** ---

Let a, b, and c be positive real numbers such that $0<a,b,c<1$ and $ab+bc+ca=1$. Prove that:

$$\frac{a}{1-a^2}+\frac{b}{1-b^2}+\frac{c}{1-c^2}\geq\frac{3\sqrt{3}}{2}$$

--- **AI3 - 81** ---

Let a, b, and c be positive real numbers such that $0<a,b,c<1$ and $ab+bc+ca=1$. Prove that:

$$\frac{a}{1-a^2}+\frac{b}{1-b^2}+\frac{c}{1-c^2}\geq\frac{3}{4}\left(\frac{1-a^2}{a}+\frac{1-b^2}{b}+\frac{1-c^2}{c}\right)$$

--- **AI3 - 82** ---

Let a, b, and c be real numbers such that $ab+bc+ca=1$ and $-1<a,b,c<1$. Prove that:

$$\sum_{cyc}\frac{a^2+b^2}{\left(1-a^2\right)\left(1-b^2\right)}\geq\frac{9}{2}$$

MathLinks, t=259913, 2009

------------------------------- **AI3 - 88** -------------------------------

For positive real numbers *a*, *b*, and *c* with $a+b+c=1$, show that

$$\frac{1+a}{1-a}+\frac{1+b}{1-b}+\frac{1+c}{1-c}\le 2\left(\frac{b}{a}+\frac{c}{b}+\frac{a}{c}\right)$$

You need not state when equality holds.

14th Japanese MO, 2004

------------------------------- **AI3 - 89** -------------------------------

a) If *a*, *b* and *c* are three real numbers, all different from 1, such that $abc=1$, then prove that

$$\frac{a^2}{(a-1)^2}+\frac{b^2}{(b-1)^2}+\frac{c^2}{(c-1)^2}\ge 1 .$$

b) Prove that equality is achieved for infinity many triples of rational numbers *a*, *b* and *c*.

Walther Janous, IMO Shortlist, 2008 & EXCALIBUR, 13(3), 2008

------------------------------- **AI3 - 90** -------------------------------

Let *a*, *b*, and *c* be positive real numbers such that $a,b,c>1$. Prove that:

$$\frac{a^4}{(b-1)^2}+\frac{b^4}{(c-1)^2}+\frac{c^4}{(a-1)^2}\ge 48$$

EXCALIBUR, 13(4) & 13(5), 2008

------------------------------- **AI3 - 91** -------------------------------

Let *a*, *b*, and *c* be positive real numbers such that $a,b,c>1$ and $ab+bc+ca=abc$. Prove that:

$$\frac{c^4+4a-4}{(a-1)^2}+\frac{a^4+4b-4}{(b-1)^2}+\frac{b^4+4c-4}{(c-1)^2}\ge 54$$

Panagiote Ligouras, MathLinks, t=298586, 2009

------------------------------- **AI3 - 92** -------------------------------

Let *a*, *b*, and *c* be non-negative real numbers such that $a+b+c=1$. Prove that:

$$3\le\frac{1}{1-ab}+\frac{1}{1-bc}+\frac{1}{1-ca}\le\frac{27}{8}$$

Sefket Arslanagic, CRUX, n. 2786

AI3 - 99

Let a, b, c be non-negative real numbers, no two of which are zero. Prove that

$$\frac{a^3}{\left(2a^2+b^2\right)\left(2a^2+c^2\right)}+\frac{b^3}{\left(2b^2+c^2\right)\left(2b^2+a^2\right)}+\frac{c^3}{\left(2c^2+a^2\right)\left(2c^2+b^2\right)}\le\frac{1}{a+b+c}$$

Vasile Cirtoaje, MathLinks, 2005

AI3 - 100

If a, b, c are non-negative real numbers, then

$$\frac{a^2-bc}{2a^2+b^2+c^2}+\frac{b^2-ca}{a^2+2b^2+c^2}+\frac{c^2-ab}{a^2+b^2+2c^2}\ge 0$$

Nguyen Anh Tuan, MathLinks, 2005

AI3 - 101

Let a, b, c be non-negative reals and let p, q be positive reals. Prove that

$$\sum_{cyc}\left[\frac{\left(a^2-bc\right)\left(p\sqrt{2a^2+b^2+c^2}+q\right)}{2a^2+b^2+c^2}\right]\ge 0$$

Panagiote Ligouras, 2009

AI3 - 102

If a, b, c are non-negative real numbers, then

$$\frac{a^2-bc}{\sqrt{2a^2+b^2+c^2}}+\frac{b^2-ca}{\sqrt{a^2+2b^2+c^2}}+\frac{c^2-ab}{\sqrt{a^2+b^2+2c^2}}\ge 0$$

Nguyen Anh Tuan, MathLinks, 2005

AI3 - 103

Let a, b, c be non-negative real numbers, no two of which are zero. Prove that

$$\frac{1}{2a^2+bc}+\frac{1}{2b^2+ca}+\frac{1}{2c^2+ab}\ge\frac{6}{a^2+b^2+c^2+ab+bc+ca}$$

AI3 - 109

Let a, b, c be positive reals such that $\sqrt{a}+\sqrt{b}+\sqrt{c}=3$. Prove that

$$\frac{a^2}{2b^2+a}+\frac{b^2}{2c^2+b}+\frac{c^2}{2a^2+c}\ge 1$$

AI3 - 110

Let a, b, c be positive reals such that $a+b+c=3$. Prove that

$$\frac{a^2}{2b^3+a}+\frac{b^2}{2c^3+b}+\frac{c^2}{2a^3+c}\ge 1,\quad a+b+c=3$$

AI3 - 111

Let a, b, c be non-negative real numbers, no two of which are zero. Then

$$\frac{a^2}{2b^2-bc+2c^2}+\frac{b^2}{2c^2-ca+2a^2}+\frac{c^2}{2a^2-ab+2b^2}\ge 1$$

Vasile Cirtoaje

AI3 - 112

Let a, b, c be positive real numbers. Prove that

$$\frac{a^3}{2a^2-ab+2b^2}+\frac{b^3}{2b^2-bc+2c^2}+\frac{c^3}{2c^2-ca+2a^2}\ge\frac{a+b+c}{3}$$

Nguyen Viet Anh

AI3 - 113

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{2a+b}+\frac{b}{2b+c}+\frac{c}{2c+a}\le 1$$

Gazeta Matematică, 2000

AI3 - 114

Let a, b, c be positive real numbers. Prove that

$$\frac{1}{2a+b}+\frac{1}{2b+c}+\frac{1}{2c+a}\ge\frac{3}{a+b+c}$$

--------------------------------- **AI3 - 121** ---------------------------------

Show that for all positive reals a, b, c,

$$\left(\frac{a+2b}{a+2c}\right)^3+\left(\frac{b+2c}{b+2a}\right)^3+\left(\frac{c+2a}{c+2b}\right)^3\geq 3$$

MOP, 2004

--------------------------------- **AI3 - 122** ---------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{a+2b+c}+\frac{b}{a+b+2c}+\frac{c}{2a+b+c}\geq\frac{3}{4}$$

--------------------------------- **AI3 - 123** ---------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{2a+b+c}+\frac{b}{a+2b+c}+\frac{c}{a+b+2c}\leq\frac{3}{4}$$

M&Y-9, 2004

--------------------------------- **AI3 - 124** ---------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{ab}{a+b+2c}+\frac{ca}{a+2b+c}+\frac{bc}{2a+b+c}\leq\frac{a+b+c}{4}$$

M&Y-3, 2005

--------------------------------- **AI3 - 125** ---------------------------------

Given three variables a, b and c satisfying the following equality $a+b+c=9a^{\frac{2}{3}}b^{\frac{2}{3}}c^{\frac{2}{3}}$ and $a,b,c\in R^+$, prove that

$$\frac{ab}{a+b+2c}+\frac{bc}{2a+b+c}+\frac{ca}{a+2b+c}\geq\frac{1}{4}$$

Shaastra Online Math Contest, 2009

--------------------------------- **AI3 - 126** ---------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{ab+c}{a+b+2c}+\frac{ca+b}{a+2b+c}+\frac{bc+a}{2a+b+c}\leq\frac{a+b+c+3}{4}$$

AI3 - 133

Let a, b, c be positive real numbers. Prove that

$$\frac{c^2+ab}{a+b+2c}+\frac{b^2+ac}{a+2b+c}+\frac{a^2+bc}{2a+b+c}\le\frac{1}{2}\left(\frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}+\frac{a+b+c}{2}\right)$$

Panagiote Ligouras, MathLinks, t=259342, 2009

AI3 - 134

Let a, b, c be positive real numbers. Prove that

$$\frac{b(c+a)^2}{a+2b+c}+\frac{c(a+b)^2}{a+b+2c}+\frac{a(b+c)^2}{2a+b+c}\ge\sqrt{3abc(a+b+c)}$$

MathLinks, t=259361, 2009

AI3 - 135

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{(b+c)(2a+b+c)}+\frac{b}{(c+a)(2b+c+a)}+\frac{c}{(a+b)(2c+a+b)}\ge\frac{9}{8(a+b+c)}$$

AI3 - 136

Let a, b, c be positive real numbers. Prove that

$$\frac{a(a+b)}{(b+c)(2a+b+c)}+\frac{b(b+c)}{(c+a)(2b+c+a)}+\frac{c(c+a)}{(a+b)(2c+a+b)}\ge\frac{3}{4}$$

AI3 - 137

Let a, b, c be positive real numbers. Prove that

$$\sum_{cyc}\frac{ab(a+c)^2(b+c)^2}{(2a+b+c)(a+2b+c)}\ge abc(a+b+c)$$

MathLinks, t=259603, 2009

AI3 - 138

Let a, b, c be positive real numbers. Prove that

$$\frac{a+b}{b+c}\cdot\frac{a}{2a+b+c}+\frac{b+c}{c+a}\cdot\frac{a}{2b+c+a}+\frac{c+a}{a+b}\cdot\frac{c}{2c+a+b}\ge\frac{3}{4}$$

D. Olteanu, Gazeta Matematică

AI3 - 145

Let a, b, and c be positive real numbers such that $abc=1$. Prove that

$$\frac{a(1+a+ab)}{(a+2b)^2}+\frac{b(1+b+bc)}{(b+2c)^2}+\frac{c(1+c+ca)}{(c+2a)^2}\ge 1$$

MathLinks, t=258676, 2009

AI3 - 146

Let a, b, and c be positive real numbers such that $abc=1$. Prove that

$$\frac{a(1+a+ab)(1+a+2b)}{(a+2b)^2}+\frac{b(1+b+bc)(1+b+2c)}{(b+2c)^2}+\frac{c(1+c+ca)(1+c+2a)}{(c+2a)^2}\ge 4$$

Panagiote Ligouras, MathLinks, t=307913, 2009

AI3 - 147

Let a, b, and c be positive real numbers. Prove that

$$\frac{2ac+ab+b^2}{(2c+b)^2}+\frac{2ab+bc+c^2}{(2a+c)^2}+\frac{2bc+ca+a^2}{(2b+a)^2}\ge\frac{4}{3}$$

Panagiote Ligouras, MathLinks, t=307910, 2009

AI3 - 148

Let a, b, and c be positive real numbers such that $abc=1$. Prove that

$$\frac{a(1+c+a+ab)+2bc}{(a+2b)^2}+\frac{b(1+a+b+bc)+2ca}{(b+2c)^2}+\frac{c(1+b+c+ca)+2ab}{(c+2a)^2}\ge 2$$

Panagiote Ligouras, ML, t=313583, 2009

AI3 - 149

Let a, b, and c be positive real numbers such that $abc=1$. Prove that

$$\sum_{cyc}\left[\frac{a(1+c+a+ab)+2bc}{(a+2b)^2}\right]+\frac{a^2+b^2+c^2}{ab+bc+ca}\ge 3$$

Panagiote Ligouras, MathLinks, t=258695, 2009

AI3 - 155

Let a, b, and c be positive real numbers. Prove that

$$\sum_{cyc}\frac{a^2+b^2}{(2c+b)^2}+2\sum_{cyc}\frac{ab}{(2a+c)(2c+b)}\geq\frac{4}{3}$$

Panagiote Ligouras, MathLinks, t= 320116, 2009

AI3 - 156

Let a, b, and c be three arbitrary real numbers. Prove that

$$\frac{1}{(2a-b)^2}+\frac{1}{(2b-c)^2}+\frac{1}{(2c-a)^2}\geq\frac{11}{7\left(a^2+b^2+c^2\right)}$$

Pham Kim Hung

AI3 - 157

Let a, b, and c be positive real numbers such that $a^2+b^2+c^2=1$. Then

$$\frac{a^2}{1+2bc}+\frac{b^2}{1+2ca}+\frac{c^2}{1+2ab}\geq\frac{3}{5}$$

Raja Oktovin, MathLinks, t=248249, 2008

AI3 - 158

Let a, b, c be non-negative real numbers such that $a+b+c=3$. Prove that

$$\sqrt{\frac{a}{1+2bc}}+\sqrt{\frac{b}{1+2ca}}+\sqrt{\frac{c}{1+2ab}}\geq\sqrt{3}$$

Pham Kim Hung

AI3 - 159

Let a, b, c be non-negative real numbers. Prove that

$$\frac{c}{\sqrt{4a^2+ab+4b^2}}+\frac{a}{\sqrt{4b^2+bc+4c^2}}+\frac{b}{\sqrt{4c^2+ca+4a^2}}\geq 1$$

Pham Kim Hung & Vo Quoc Ba Can

AI3 - 160

Let a, b, and c be positive real numbers. Prove that

$$\frac{a}{\sqrt{4a^2+ab+4b^2}}+\frac{b}{\sqrt{4b^2+bc+4c^2}}+\frac{c}{\sqrt{4c^2+ca+4a^2}}\leq 1$$

Bin Zhao, Mathematical Reflections, n. O32, 2006

AI3 - 167

Let a, b, and c be non-negative numbers such that $ab+bc+ac\neq 0$. Then

$$\frac{a+b}{4c^2+ab}+\frac{b+c}{4a^2+bc}+\frac{a+c}{4b^2+ca}\geq\frac{6(a+b+c)}{5(ab+bc+ac)}$$

Michael Rozenberg, MathLinks, t=262789, 2009

AI3 - 168

Let a, b, and c be positive real numbers such that $a+b+c=3$. Prove that

$$\sum_{cyc}\frac{1}{\sqrt{4a^2+bc+7a+8}}\geq\frac{2}{3}$$

Panagiote Ligouras, MathLinks, t=310812, 2009

AI3 - 169

Let a, b, and c be positive real numbers. Prove that

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\geq\frac{a+b+c}{abc}$$

AI3 - 170

Let a, b, c be positive real numbers such that $a+b+c=3$. Prove that

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\geq a^2+b^2+c^2$$

Romania-TST, Iasi & Bucharest, 2006

AI3 - 171

Suppose that a, b, and c are positive reals satisfying $a^2+b^2+c^2=1$. Prove that

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\geq 3+\frac{2\left(a^3+b^3+c^3\right)}{abc}$$

Ho-joo Lee, CRUX, n. 2532

AI3 - 172

Let a, b, c be positive real numbers such that $a+b+c=1$. Prove that

$$abc\left(1+\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\geq\frac{28}{27}$$

Walther Janous, CRUX, Klamkin-02

AI3 - 179

Let a, b, and c be positive real numbers. Prove that

$$\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}\geq\frac{9}{4}+\left(\frac{a+b}{c}+\frac{c+a}{b}+\frac{b+c}{a}\right)-\frac{9(ab+bc+ca)}{4(a^2+b^2+c^2)}$$

Panagiote Ligouras, ML, t=313586, 2009

AI3 - 180

Let a, b, and c be positive real numbers. Prove that

$$\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}\geq\frac{10}{3}\cdot\frac{a^2+b^2+c^2}{ab+bc+ca}-3\cdot\frac{ab+bc+ca}{a^2+b^2+c^2}-1$$

Panagiote Ligouras, ML, t=320792, 2009

AI3 - 181

Let a, b, and c be positive real numbers. Prove that

$$\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}\geq 18-\frac{120abc}{(a+b)(b+c)(c+a)}$$

Tran Quoc Anh, ML t=293216, 2009

AI3 - 182

Let a, b, and c be positive real numbers. Prove that

$$\frac{1}{2}\left(\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}+9\right)\geq\frac{a}{b}+2\cdot\sqrt{\frac{b}{c}}+3\cdot\sqrt[3]{\frac{c}{a}}$$

AI3 - 183

Let a, b, and c be positive real numbers. Prove that

$$\frac{3}{2}\left(\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}+1\right)\geq\frac{a}{b}+2\cdot\sqrt{\frac{b}{c}}+3\cdot\sqrt[3]{\frac{c}{a}}\geq 6$$

AI3 - 184

Let a, b, and c be positive real numbers. Prove that

$$\frac{a}{b^2}+\frac{b}{c^2}+\frac{c}{a^2}\leq\frac{a^2}{b^3}+\frac{b^2}{c^3}+\frac{c^2}{a^3}$$

------------------------------- **AI3 - 191** -------------------------------

Let a, b, and c be positive real numbers such that $abc=1$. Prove that

$$\frac{c}{a^2(1+b)}+\frac{a}{b^2(1+c)}+\frac{b}{c^2(1+a)}\geq\frac{3}{2}$$

------------------------------- **AI3 - 192** -------------------------------

Let x, y, and z be positive real numbers. Prove that

$$\left(2+\frac{xy}{z^2}\right)^2+\left(2+\frac{yz}{x^2}\right)^2+\left(2+\frac{zx}{y^2}\right)^2\geq\frac{9\left(x^2z+z^2y+y^2x\right)^2}{xyz\left(x^2y+y^2z+z^2x\right)}$$

Panagiote Ligouras, MathLinks, t=310024, 2009

------------------------------- **AI3 - 193** -------------------------------

Let a, b, and c be positive real numbers such that $a+b+c=1$. Prove that

$$\left(\frac{1}{a^2}-1\right)\left(\frac{1}{b^2}-1\right)\left(\frac{1}{c^2}-1\right)\geq 2^9$$

------------------------------- **AI3 - 194** -------------------------------

Let a, b, and c be non-negative real numbers such that $ab+bc+ca=3$. Prove that

$$\frac{1}{1+a^2}+\frac{1}{1+b^2}+\frac{1}{1+c^2}\geq\frac{3}{2}$$

Vasile Cirtoaje, MathLinks, 2005

------------------------------- **AI3 - 195** -------------------------------

If $a,b,c\leq 1$ and $a+b+c=1$. Prove that

$$\frac{1}{1+a^2}+\frac{1}{1+b^2}+\frac{1}{1+c^2}\leq\frac{27}{10}$$

Titu Andreescu & Gabriel Dospinescu

------------------------------- **AI3 - 196** -------------------------------

Let a, b, and c be positive real numbers such that $a^2+b^2+c^2=1$. Then

$$\frac{a}{b^2+1}+\frac{b}{c^2+1}+\frac{c}{a^2+1}\geq\frac{3}{4}\left(a\sqrt{a}+b\sqrt{b}+c\sqrt{c}\right)^2$$

Hellenic-IMO-TST, 2002

------------------------------- **AI3 - 203** -------------------------------

Let a, b, and c be positive reals such that $ab+bc+ca=1$. Prove that

$$\frac{ab}{a^2+1}+\frac{bc}{b^2+1}+\frac{ca}{c^2+1}\le\frac{1}{16}\left[\frac{a}{b}+\frac{b}{c}+\frac{c}{a}+2\left(\frac{b}{a}+\frac{c}{b}+\frac{a}{c}\right)+3\right]$$

MathLinks, t=259357, 2009

------------------------------- **AI3 - 204** -------------------------------

Let a, b, and c be positive real numbers with $ab+bc+ca=3$. Prove that

$$\frac{ab}{c^2+1}+\frac{bc}{a^2+1}+\frac{ca}{b^2+1}\ge\frac{3}{2}$$

Titu Zvonaru, Mathematical Mayhem, 34(4) M299, 2008

------------------------------- **AI3 - 205** -------------------------------

Let a, b, and c be positive real numbers such that $a+b+c=3$. Prove that

$$\frac{a^2}{b^2+1}+\frac{b^2}{c^2+1}+\frac{c^2}{a^2+1}\ge\frac{3}{2}$$

Faruk Zejnulahi & Sefket Arslanagic, CRUX, n. 2994

------------------------------- **AI3 - 206** -------------------------------

Let a, b, and c be positive real numbers such that $a+b+c=3$. Prove that

$$\frac{1+a}{1+b^2}+\frac{1+b}{1+c^2}+\frac{1+c}{1+a^2}\ge 3$$

------------------------------- **AI3 - 207** -------------------------------

Let a, b, and c be positive real numbers such that $a+b+c=1$. Prove that

$$\frac{1+a^2}{1+b^2}+\frac{1+b^2}{1+c^2}+\frac{1+c^2}{1+a^2}\le\frac{7}{2}$$

MathLinks, t=260118, 2009

------------------------------- **AI3 - 208** -------------------------------

Let a, b, and c be positive real numbers such that $ab+bc+ca=1$. Prove that

$$\frac{1-a^2}{1+a^2}+\frac{1-b^2}{1+b^2}+\frac{1-c^2}{1+c^2}\le\frac{3}{2}$$

Gazeta Matematică, 2003

AI3 - 215

Let a, b, and c be positive real numbers. Prove that

$$6\prod_{cyc}\left(\frac{a^3+1}{a^2+1}\right)\geq \max\left\{\sum_{cyc}\frac{a(1+bc)(a^2+1)}{a^3+1},\sum_{cyc}\frac{ab(1+c)(a^2b^2+1)}{a^3b^3+1}\right\}$$

Mihàly Bencze, CRUX, 34(4) n. 3349, 2008

AI3 - 216

Let a, b, and c be positive real numbers such that $a+b+c=abc$. Then

$$\frac{1}{\sqrt{a^2+1}}+\frac{1}{\sqrt{b^2+1}}+\frac{1}{\sqrt{c^2+1}}\leq\frac{3}{2}$$

EXCALIBUR, 6(2) & 6(3), 2001

AI3 - 217

Let r be a real number, $0<r\leq 1$, and let a, b, and c be positive real numbers such that $abc=r^3$. Prove that

$$\frac{1}{\sqrt{a^2+1}}+\frac{1}{\sqrt{b^2+1}}+\frac{1}{\sqrt{c^2+1}}\leq\frac{3}{\sqrt{1+r^2}}$$

Arkady Alt, CRUX, 34(3), n. 3329, 2008

AI3 - 218

Let a, b, and c be positive reals such that $ab+bc+ca=1$. Prove that

$$\frac{a}{\sqrt{a^2+1}}+\frac{b}{\sqrt{b^2+1}}+\frac{c}{\sqrt{c^2+1}}\leq\frac{3}{2}$$

AI3 - 219

Let a, b, and c be positive real numbers such that $a+b+c=abc$. Then

$$\frac{a}{\sqrt{a^2+1}}+\frac{b}{\sqrt{b^2+1}}+\frac{c}{\sqrt{c^2+1}}\leq\frac{3\sqrt{3}}{2}$$

------------------------------- **AI3 - 225** -------------------------------

Let a, b, and c be non-negative reals such that $ab+bc+ca=3$. Prove that

$$\frac{1}{a^2+2}+\frac{1}{b^2+2}+\frac{1}{c^2+2}\le 1$$

------------------------------- **AI3 - 226** -------------------------------

Let a, b, and c be non-negative reals such that $abc=1$. Prove that

$$\frac{7-6a}{a^2+2}+\frac{7-6b}{b^2+2}+\frac{7-6c}{c^2+2}\ge 1$$

Vasile Cirtoaje, MathLinks, t=245046, 2008

------------------------------- **AI3 - 227** -------------------------------

Let a, b, and c be positive real numbers. Prove that

$$\frac{1}{a^2+2bc}+\frac{1}{b^2+2ca}+\frac{1}{c^2+2ab}\ge \frac{2}{ab+bc+ca}$$

Vasile Cirtoaje, MathLinks, 2005

------------------------------- **AI3 - 228** -------------------------------

Let a, b, and c be positive real numbers. Prove that

$$\frac{a^2}{a^2+2bc}+\frac{b^2}{b^2+2ca}+\frac{c^2}{c^2+2ab}\ge 1$$

Romania, 1997

------------------------------- **AI3 - 229** -------------------------------

Let a, b, and c be positive real numbers. Prove that

$$\frac{bc}{a^2+2bc}+\frac{ca}{b^2+2ca}+\frac{ab}{c^2+2ab}\le 1$$

Romania, 1997

------------------------------- **AI3 - 230** -------------------------------

Let a, b, and c be positive real numbers. Prove that

$$\frac{2a^2-bc}{a^2+2bc}+\frac{2b^2-ca}{b^2+2ca}+\frac{2c^2-ab}{c^2+2ab}\ge 1$$

Panagiote Ligouras, MathLinks, t=310029, 2009

AI3 - 237

Let a, b, and c be positive real numbers. Prove that

$$\frac{a}{\sqrt{a^2+2bc}}+\frac{b}{\sqrt{b^2+2ca}}+\frac{c}{\sqrt{c^2+2ab}} \le \frac{a+b+c}{\sqrt{ab+bc+ca}}$$

Ho Phu Thai & Da Nang, Mathematical Reflections, n. O39, 2007

AI3 - 238

Prove that for all non-negative real numbers a, b, c, we have

$$\sqrt{\frac{2a^2+bc}{a^2+2bc}}+\sqrt{\frac{2b^2+ca}{b^2+2ca}}+\sqrt{\frac{2c^2+ab}{c^2+2ab}} \ge 2\sqrt{2}$$

Pham Kim Hung

AI3 - 239

Let a, b, and c be non-negative real numbers. Prove that

$$\frac{a}{a^2+2bc}+\frac{b}{b^2+2ca}+\frac{c}{c^2+2ab} \le \frac{a+b+c}{ab+bc+ca}$$

Tran Quoc Anh, MathLinks, t=224981, 2008

AI3 - 240

Let a, b, and c be positive real numbers. Prove that

$$\frac{a-1}{a^2+2bc}+\frac{b-1}{b^2+2ca}+\frac{c-1}{c^2+2ab} \le \frac{a+b+c-2}{ab+bc+ca}$$

Panagiote Ligouras, ML, t=322098, 2010

AI3 - 241

Let a, b, and c be positive real numbers. Prove or disprove that

$$\sum_{cyc}\frac{a\left(\sqrt{a^2+2bc}+1\right)-1}{a^2+2bc} \le \frac{(a+b+c)\left(\sqrt{ab+bc+ca}+1\right)-2}{ab+bc+ca}$$

Panagiote Ligouras, MathLinks, t=251776, 2009

AI3 - 242

Let a, b, and c be positive real numbers. Prove that

$$\frac{1}{a^2+3}+\frac{1}{b^2+3}+\frac{1}{c^2+3} \le \frac{a+b+c}{4\sqrt{abc}}$$

Romania, 2003

AI3 - 248

Let a, b, and c be positive real numbers such that $abc = 1$. Prove that

$$\frac{a^3}{a^2+4ab+b^2}+\frac{b^3}{b^2+4bc+c^2}+\frac{c^3}{c^2+4ca+a^2}\geq 1$$

José Luis Dìaz-Barrero, SSMJ, n. 5054, 2009

AI3 - 249

Let a, b, and c be positive real numbers such that $abc \leq 1$. Prove that

$$\frac{a}{b^2+b}+\frac{b}{c^2+c}+\frac{c}{a^2+a}\geq \frac{3}{2}$$

Cao Minh Quang, CRUX, 35(2), n. 3421, 2009

AI3 - 250

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{1}{a^2+ab+b^2}+\frac{1}{b^2+bc+c^2}+\frac{1}{c^2+ca+a^2}\geq \frac{9}{(a+b+c)^2}$$

Vasile Cirtoaje, Gazeta Matematică-B, 9, 2000

AI3 - 251

Let a, b, and c be positive real numbers such that $a+b+c=3$. Prove that

$$\frac{1}{a^2+ab+b^2}+\frac{1}{b^2+bc+c^2}+\frac{1}{c^2+ca+a^2}\leq \frac{1}{abc}$$

AI3 - 252

Let a, b, and c be positive real numbers. Prove that

$$\frac{a^2}{b\left(a^2+ab+b^2\right)}+\frac{b^2}{c\left(b^2+bc+c^2\right)}+\frac{c^2}{a\left(c^2+ca+a^2\right)}\geq \frac{3}{a+b+c}$$

AI3 - 253

Let a, b, and c be positive real numbers such that $a+b+c=3$. Prove that

$$\frac{a^2+b}{b\left(a^2+ab+b^2\right)}+\frac{b^2+c}{c\left(b^2+bc+c^2\right)}+\frac{c^2+a}{a\left(c^2+ca+a^2\right)}\geq 2$$

Panagiote Ligouras, ML, t=322100, 2010

AI3 - 260

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{a(b+c)}{b^2+bc+c^2}+\frac{b(c+a)}{c^2+ca+a^2}+\frac{c(a+b)}{a^2+ab+b^2}\geq 2$$

AI3 - 261

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{a^2+bc}{b^2+bc+c^2}+\frac{b^2+ca}{c^2+ca+a^2}+\frac{c^2+ab}{a^2+ab+b^2}\geq 2$$

Vasile Cirtoaje, ML, 2005

AI3 - 262

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{2a^2+3bc}{b^2+bc+c^2}+\frac{2b^2+3ca}{c^2+ca+a^2}+\frac{2c^2+3ab}{a^2+ab+b^2}\geq 5$$

AI3 - 263

Let a, b, and c be positive real numbers. Prove that

$$\frac{5a^2+2bc}{b^2+bc+c^2}+\frac{5b^2+2ca}{c^2+ca+a^2}+\frac{5c^2+2ab}{a^2+ab+b^2}\geq 7$$

Panagiote Ligouras, ML, t=322104, 2010

AI3 - 264

Let a, b, and c be positive real numbers and p, q >0. Prove that

$$\frac{pa^2+qbc}{b^2+bc+c^2}+\frac{pb^2+qca}{c^2+ca+a^2}+\frac{pc^2+qab}{a^2+ab+b^2}\geq p+q$$

Panagiote Ligouras, 2009

AI3 - 265

Let a, b, and c be positive real numbers. Prove that

$$\frac{a^2+b^2-3c^2}{a^2+ab+b^2}+\frac{b^2+c^2-3a^2}{b^2+bc+c^2}+\frac{c^2+a^2-3b^2}{c^2+ca+a^2}+\frac{4(ab+bc+ca)}{3(a^2+b^2+c^2)}\leq\frac{1}{3}$$

Panagiote Ligouras, MathLinks, t=322515, 2010

AI3 - 271

Let a, b, and c be positive real numbers. Prove that

$$\frac{a^3+b^3}{a^2+ab+b^2}+\frac{b^3+c^3}{b^2+bc+c^2}+\frac{c^3+a^3}{c^2+ca+a^2}\geq\frac{2}{3}(a+b+c)$$

AI3 - 272

Let a, b, c be positive real numbers and $k > 0$. Prove that

$$\frac{ka^3+b^3}{a^2+ab+b^2}+\frac{kb^3+c^3}{b^2+bc+c^2}+\frac{kc^3+a^3}{c^2+ca+a^2}\geq\frac{k+1}{3}(a+b+c)$$

Panagiote Ligouras, 2009

AI3 - 273

Let a, b, and c be positive real numbers such that $abc=1$. Prove that

$$\frac{a^3+b^3}{a^2+ab+b^2}+\frac{b^3+c^3}{b^2+bc+c^2}+\frac{c^3+a^3}{c^2+ca+a^2}\geq 2$$

AI3 - 274

Let a, b, and c be positive real numbers such that $a+b+c=3$. Prove that

$$\frac{a^3}{a^2+ab+b^2}+\frac{b^3}{b^2+bc+c^2}+\frac{c^3}{c^2+ca+a^2}\geq 1$$

AI3 - 275

Let a, b, and c be positive real numbers such that $abc=1$. Prove that

$$\frac{a^3}{a^2+ab+b^2}+\frac{b^3}{b^2+bc+c^2}+\frac{c^3}{c^2+ca+a^2}\geq 1$$

José Luis Diaz-Barrero, SSMJ 2009(2) n. 5054

AI3 - 276

Let a, b, c be positive real numbers such that $abc=1$ and $k > 0$. Prove that

$$\frac{ka^3+b^3}{a^2+ab+b^2}+\frac{kb^3+c^3}{b^2+bc+c^2}+\frac{kc^3+a^3}{c^2+ca+a^2}\geq k+1$$

Panagiote Ligouras, 2009

AI3 - 282

Let a, b, and c be positive real numbers. Prove that

$$\sqrt{\frac{a^3}{a^2+ab+b^2}}+\sqrt{\frac{b^3}{b^2+bc+c^2}}+\sqrt{\frac{c^3}{c^2+ca+a^2}}\ge\frac{\sqrt{a}+\sqrt{b}+\sqrt{c}}{\sqrt{3}}$$

Le Trung Kien

AI3 - 283

If $a, b, c > 0$, prove that

$$\frac{ab+bc+ca}{\sqrt{a^2+ab+b^2}+\sqrt{b^2+bc+c^2}+\sqrt{c^2+ca+a^2}}\le\frac{a+b+c}{3\sqrt{3}}$$

Murray S. Klamkin, CRUX n. 805

AI3 - 284

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{1}{b^2+c^2}+\frac{1}{c^2+a^2}+\frac{1}{a^2+b^2}\ge\frac{15}{2\left(a^2+b^2+c^2+ab+ac+ca\right)}$$

AI3 - 285

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{1}{b^2+c^2}+\frac{1}{c^2+a^2}+\frac{1}{a^2+b^2}\ge\frac{10}{\left(a+b+c\right)^2}$$

AI3 - 286

Let a, b, and c be positive real numbers such that $a+b+c=2$. Prove that

$$\frac{1}{b^2+c^2}+\frac{1}{c^2+a^2}+\frac{1}{a^2+b^2}\ge\frac{5}{2}$$

AI3 - 287

Let a, b, c be non-negative real numbers. Prove that

$$\frac{1}{a^2+b^2}+\frac{1}{b^2+c^2}+\frac{1}{c^2+a^2}\ge\frac{6}{ab+bc+ca}-\frac{8}{a^2+b^2+c^2}$$

Pham Kim Hung

---------------------------------- **AI3 - 294** ----------------------------------

Let a, b, and c be positive real numbers such that $abc=1$. Then

$$\frac{a}{a^2+b^2}+\frac{b}{b^2+c^2}+\frac{c}{c^2+a^2}\le\frac{a^3+b^3+c^3+15}{12}$$

Panagiote Ligouras, ML, t=322517, 2010

---------------------------------- **AI3 - 295** ----------------------------------

Let a, b, and c be positive real numbers. Prove that

$$\frac{a+b}{a^2+b^2}+\frac{b+c}{b^2+c^2}+\frac{c+a}{c^2+a^2}\le\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$$

---------------------------------- **AI3 - 296** ----------------------------------

Let a, b, and c be positive real numbers. Prove that

$$\frac{a-b}{b^2+c^2}+\frac{b-c}{c^2+a^2}+\frac{c-a}{a^2+b^2}\ge\frac{1}{10}\left(\frac{8a-5b-5c}{a(b+c)}+\frac{8b-5c-5a}{b(c+a)}+\frac{8c-5a-5b}{c(a+b)}\right)$$

Panagiote Ligouras, MathLinks, t=326767, 2010

---------------------------------- **AI3 - 297** ----------------------------------

If $c\ge b\ge a>0$, then

$$\frac{ac}{a^2+b^2}+\frac{ba}{b^2+c^2}+\frac{cb}{c^2+a^2}\le\frac{2}{3}\left(\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\right)^2$$

Panagiote Ligouras, MathLinks, t=322519, 2010

---------------------------------- **AI3 - 298** ----------------------------------

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{a(2a+b+c)}{b^2+c^2}+\frac{b(a+2b+c)}{c^2+a^2}+\frac{c(a+b+2c)}{a^2+b^2}\ge 6$$

---------------------------------- **AI3 - 299** ----------------------------------

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{(-a+b+c)^2}{b^2+c^2}+\frac{(a-b+c)^2}{c^2+a^2}+\frac{(a+b-c)^2}{a^2+b^2}\ge\frac{3}{2}$$

Vasile Cirtoaje, ML, t=259748, 2009

---------------------------------- **AI3 - 306** ----------------------------------

Let a, b, and c be positive real numbers. Prove that

$$\frac{a^3}{a^2+b^2}+\frac{b^3}{b^2+c^2}+\frac{c^3}{c^2+a^2}\le\frac{1}{2}\left(\frac{a^3}{b^2}+\frac{b^3}{c^2}+\frac{c^3}{a^2}\right)$$

Panagiote Ligouras, MathLinks, t=311797, 2009

---------------------------------- **AI3 - 307** ----------------------------------

Let a, b, and c be positive real numbers. Prove that

$$\left(\sum_{cyc}\frac{a^2}{b^2+c^2}\right)\left(\prod_{cyc}\frac{a^2+b^2}{b^2}\right)\ge\left(\sum_{cyc}\frac{a}{b+c}\right)\left(\prod_{cyc}\frac{a+b}{b}\right)$$

Panagiote Ligouras, ML, t=251794, 2009

---------------------------------- **AI3 - 308** ----------------------------------

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{2a^2+bc}{b^2+c^2}+\frac{2b^2+ac}{c^2+a^2}+\frac{2c^2+ab}{a^2+b^2}\ge\frac{9}{2}$$

---------------------------------- **AI3 - 309** ----------------------------------

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{a^2+16bc}{b^2+c^2}+\frac{b^2+16ac}{c^2+a^2}+\frac{c^2+16ab}{a^2+b^2}\ge 10$$

Vasile Cirtoaje, ML, 2005

---------------------------------- **AI3 - 310** ----------------------------------

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{ab-bc+ca}{b^2+c^2}+\frac{bc-ca+ab}{c^2+a^2}+\frac{ca-ab+bc}{a^2+b^2}\ge\frac{3}{2}$$

---------------------------------- **AI3 - 311** ----------------------------------

Let a, b, c be non-negative reals, no two of which are zero. Then

$$\frac{ab+4bc+ca}{b^2+c^2}+\frac{bc+4ca+ab}{c^2+a^2}+\frac{ca+4ab+bc}{a^2+b^2}\ge 4$$

AI3 - 318

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\sqrt{\frac{a^2+2bc}{b^2+c^2}}+\sqrt{\frac{b^2+2ac}{c^2+a^2}}+\sqrt{\frac{c^2+2ab}{a^2+b^2}}\ge 3$$

Vo Quoc Ba Can & Vu Dinh Quy

AI3 - 319

Let a, b, c be positive real numbers. Prove that

$$\sqrt{\frac{a^2+4bc}{b^2+c^2}}+\sqrt{\frac{b^2+4ac}{c^2+a^2}}+\sqrt{\frac{c^2+4ab}{a^2+b^2}}\ge 2+\sqrt{2}$$

Vo Quoc Ba Can, CRUX 35(4) n. 3419, 2009

AI3 - 320

Let a, b, c be positive real numbers. Prove that

$$\sum_{cyc}\left(\frac{2\sqrt{a^2+2bc}+\sqrt{a^2+4bc}}{\sqrt{b^2+c^2}}\right)\ge 8+\sqrt{2}$$

Panagiote Ligouras, 2009

AI3 - 321

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{\sqrt{\left(a^2+b^2\right)\left(a^2+c^2\right)}}+\frac{b}{\sqrt{\left(b^2+a^2\right)\left(b^2+c^2\right)}}+\frac{c}{\sqrt{\left(c^2+a^2\right)\left(c^2+b^2\right)}}\le\frac{a^2+b^2+c^2}{2abc}$$

Romania, local MO, 1988

AI3 - 322

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\sqrt[3]{\frac{a^2+bc}{b^2+c^2}}+\sqrt[3]{\frac{b^2+ac}{c^2+a^2}}+\sqrt[3]{\frac{c^2+ab}{a^2+b^2}}\ge\frac{9\sqrt[3]{abc}}{a+b+c}$$

Pham Huu Duc, Mathematical Reflections, n S27, 2006

--------------------------------- **AI3 - 329** ---------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{3\left(a^2+b^2+c^2\right)}{4abc} > \frac{1}{\sqrt{a^2+b^2}} + \frac{1}{\sqrt{b^2+c^2}} + \frac{1}{\sqrt{c^2+a^2}} > \frac{2(a+b+c)}{a^2+b^2+c^2}$$

Zdravko F. Starc, RSME n.128, 2009

--------------------------------- **AI3 - 330** ---------------------------------

Let a, b, c be positive real numbers. Prove that

$$3\left(\frac{a^2+b^2+c^2}{4abc} + \sqrt{2}\right) > \frac{2a+1}{\sqrt{a^2+b^2}} + \frac{2b+1}{\sqrt{b^2+c^2}} + \frac{2c+1}{\sqrt{c^2+a^2}} > 2\left(\frac{a+b+c}{a^2+b^2+c^2} + 1\right)$$

Panagiote Ligouras, MathLinks, t=326771, 2010

--------------------------------- **AI3 - 331** ---------------------------------

Let a, b, c be positive real numbers such that $a^2+b^2+c^2=3$. Prove that

$$\sqrt{\frac{a}{a^2+b^2+1}} + \sqrt{\frac{b}{b^2+c^2+1}} + \sqrt{\frac{c}{c^2+a^2+1}} \le \sqrt{3}$$

Pham Kim Hung

--------------------------------- **AI3 - 332** ---------------------------------

Let a, b, c be positive real numbers such that $abc=1$. Prove that

$$\frac{a^2+b^2}{a^2+b^2+1} + \frac{b^2+c^2}{b^2+c^2+1} + \frac{c^2+a^2}{c^2+a^2+1} \ge \frac{a+b}{a^2+b^2+1} + \frac{b+c}{b^2+c^2+1} + \frac{c+a}{c^2+a^2+1}$$

Jingjun Han, Mathematical Reflections n. J104, 2008

--------------------------------- **AI3 - 333** ---------------------------------

Let a, b, c be positive real numbers such that $a+b+c=3$. Prove that

$$\frac{a}{b^2+c} + \frac{b}{c^2+a} + \frac{c}{a^2+b} \ge \frac{3}{2}$$

Pham Kim Hung

--------------------------------- **AI3 - 334** ---------------------------------

Let a, b, c be positive reals such that $a+b+c=3$ and $abc=1$. Prove that

$$\frac{ab}{\left(a^2+b\right)\left(a+b^2\right)} + \frac{bc}{\left(b^2+c\right)\left(b+c^2\right)} + \frac{ca}{\left(c^2+a\right)\left(c+a^2\right)} \le \frac{3}{2}$$

AI3 - 341

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{a^2+bc}+\frac{b}{b^2+ca}+\frac{c}{c^2+ab}\le\frac{1}{2}\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)$$

Baltic way, 2003

AI3 - 342

Let a, b, c be positive real numbers. Prove that

$$\frac{bc}{a^2+bc}+\frac{ca}{b^2+ca}+\frac{ab}{c^2+ab}\le\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$$

Pham Huu Duc, Mathematical Reflections n. J60, 2007

AI3 - 343

Let a, b, c be positive real numbers. Prove that

$$\frac{a^2}{a^2+bc}+\frac{b^2}{b^2+ca}+\frac{c^2}{c^2+ab}\le 2$$

Canada

AI3 - 344

Let a, b, and c be positive real numbers. Prove that

$$\frac{a^2}{a^2+bc}+\frac{b^2}{b^2+ca}+\frac{c^2}{c^2+ab}\le\frac{a+b+c}{2\sqrt[3]{abc}}$$

Pham Huu Duc, CRUX 34(6) n. 3374, 2008

AI3 - 345

Suppose that a, b, and c are positive real numbers. Prove that

$$\frac{b+c}{a^2+bc}+\frac{c+a}{b^2+ca}+\frac{a+b}{c^2+ab}\le\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$$

Ho-Joo Lee, CRUX n. 2580

AI3 - 346

Let a, b, and c be positive real numbers. Prove that

$$\frac{a^2+3a}{a^2+bc}+\frac{b^2+3b}{b^2+ca}+\frac{c^2+3c}{c^2+ab}\le\frac{\left(a^2+b^2+c^2\right)+\left(a+b+c\right)^2+9\left(a+b+c\right)}{a^2+b^2+c^2+ab+bc+ca}$$

Panagiote Ligouras, MathLinks, t=329754, 2010

------------------------------- **AI3 - 353** -------------------------------

Let a, b, and c be positive real numbers. Prove that

$$\frac{b^2+bc+c^2}{a^2+bc}+\frac{c^2+ca+a^2}{b^2+ca}+\frac{a^2+ab+b^2}{c^2+ab}\geq\frac{21}{4}-\frac{a^3+b^3+c^3}{4abc}$$

Panagiote Ligouras, ML, t=249913, 2009

------------------------------- **AI3 - 354** -------------------------------

Let a, b, and c be positive real numbers. Prove that

$$\sum_{cyc}\frac{3b^2+4bc+3c^2}{a^2+bc}\geq\frac{33}{2}-\frac{a^3+b^3+c^3}{2abc}$$

Panagiote Ligouras, ML, t=329755, 2010

------------------------------- **AI3 - 355** -------------------------------

Let a, b, and c be positive real numbers. Prove that

$$\frac{\sqrt{ab}}{c^2+ab}+\frac{\sqrt{bc}}{a^2+bc}+\frac{\sqrt{ca}}{b^2+ca}\geq\frac{9\left(a^3+b^3+c^3\right)}{2\left(a+b+c\right)^4}$$

MIC Shortlist, ML, t=259607, 2009

------------------------------- **AI3 - 356** -------------------------------

Let a, b, and c be non-negative real numbers. Prove that

$$\frac{1}{\sqrt{a^2+bc}}+\frac{1}{\sqrt{b^2+ca}}+\frac{1}{\sqrt{c^2+ab}}\geq\frac{6}{a+b+c}$$

Pham Kim Hung

------------------------------- **AI3 - 357** -------------------------------

Let a, b, and c be non-negative real numbers. Prove that

$$\frac{1}{\sqrt{a^2+bc}}+\frac{1}{\sqrt{b^2+ca}}+\frac{1}{\sqrt{c^2+ab}}\geq\frac{2\sqrt{2}}{\sqrt{ab+bc+ca}}$$

Pham Kim Hung

------------------------------- **AI3 - 358** -------------------------------

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{a^2-bc}{\sqrt{a^2+bc}}+\frac{b^2-ca}{\sqrt{b^2+ca}}+\frac{c^2-ab}{\sqrt{c^2+ab}}\geq 0$$

Vasile Cirtoaje, ML, 2005

AI3 - 365

Let a, b, and c be positive real numbers such that $a,b,c \geq -1$. Prove that

$$\frac{a^2+1}{c^2+b+1}+\frac{b^2+1}{a^2+c+1}+\frac{c^2+1}{b^2+a+1}\geq 2$$

Laurentiu Panaitopol, JBMO, 2003

AI3 - 366

Let a, b, c be real numbers such that $-1 \leq a,b,c \leq 1$ and $a+b+c=0$. Prove that

$$\sqrt{a^2+c+1}+\sqrt{b^2+a+1}+\sqrt{c^2+b+1}\geq 3$$

Pham Thanh Nam

AI3 - 367

Prove that if the real numbers a, b and c satisfy $a^2+b^2+c^2=3$ then

$$\frac{a^2}{c^2+b+2}+\frac{b^2}{a^2+c+2}+\frac{c^2}{b^2+a+2}\geq \frac{(a+b+c)^2}{12}.$$

When does the equality hold?

Baltic Way, 2008

AI3 - 368

Let a, b, c be non-negative real numbers such that $a+b+c=3$. Prove that

$$\left(a^2-ab+b^2\right)\left(b^2-bc+c^2\right)\left(c^2-ca+a^2\right)\leq 12$$

Pham Kim Hung, ML, 2006

AI3 - 369

Let a, b, c be non-negative real numbers such that $a^2+b^2+c^2=2$. Then

$$8\left(a^2-ab+b^2\right)\left(b^2-bc+c^2\right)\left(c^2-ca+a^2\right)\leq 1$$

Pham Kim Hung

AI3 - 370

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{1}{b^2-bc+c^2}+\frac{1}{c^2-ca+a^2}+\frac{1}{a^2-ab+b^2}\geq \frac{12}{(a+b+c)^2}$$

Vasile Cirtoaje, 2006

AI3 - 377

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\frac{3a^2-bc}{b^2-bc+c^2}+\frac{3b^2-ca}{c^2-ca+a^2}+\frac{3c^2-ab}{a^2-ab+b^2}\geq 5$$

Panagiote Ligouras, ML, t=319775, 2009

AI3 - 378

Let a, b, and c be positive real numbers such that $a^4+b^4+c^4=3$. Then

$$\frac{a^3}{b^2-bc+c^2}+\frac{b^3}{c^2-ca+a^2}+\frac{c^3}{a^2-ab+b^2}\leq\frac{3}{abc}$$

AI3 - 379

Let a, b, and c be positive real numbers. Prove that

$$\frac{a^3}{b^2-bc+c^2}+\frac{b^3}{c^2-ca+a^2}+\frac{c^3}{a^2-ab+b^2}\geq\frac{3(ab+bc+ca)}{a+b+c}$$

Sefket Arslanagic, CRUX n. 2927

AI3 - 380

Let a, b, c be non-negative reals, no two of which are zero. Prove that

$$\sum_{cyc}\frac{a^3}{b^2-bc+c^2}\geq\sum_{cyc}a\geq 4\sum_{cyc}\frac{ab}{a+b+2c}$$

AI3 - 381

Let a, b, and c be positive real numbers such that $a^4+b^4+c^4=3$. Then

$$\frac{a^3+b}{b^2-bc+c^2}+\frac{b^3+c}{c^2-ca+a^2}+\frac{c^3+a}{a^2-ab+b^2}\leq\frac{3+ab+bc+ca}{abc}$$

Panagiote Ligouras, MathLinks, t=329119, 2010

AI3 - 382

Let a, b, c be non-negative reals, no two of which are zero, such that $a+b+c=2$. Prove that

$$\frac{a^3+8}{b^2-bc+c^2}+\frac{b^3+8}{c^2-ca+a^2}+\frac{c^3+8}{a^2-ab+b^2}\geq 26$$

Panagiote Ligouras, ML, t=316360, 2009

AI3 - 389

Let a, b, and c be positive real numbers such that $abc=1$. Then

$$\frac{1+ab^2}{c^3}+\frac{1+bc^2}{a^3}+\frac{1+ca^2}{b^3}\geq\frac{18}{a^3+b^3+c^3}$$

Hong Kong, 2000

AI3 - 390

Let a, b, and c be positive real numbers. Prove that

$$\frac{a}{b^3}+\frac{b}{c^3}+\frac{c}{a^3}\geq\frac{27}{\left(a+b+c\right)^2}$$

AI3 - 391

Let a, b, and c be positive real numbers such that $a^2+b^2+c^2=3$. Then

$$\frac{a^2}{b^3}+\frac{b^2}{c^3}+\frac{c^2}{a^3}\geq 3$$

MathLinks, t=259844, 2009

AI3 - 392

Let a, b, c be positive real numbers such that $a^k+b^k+c^k=3$, $k>0$. Then

$$\frac{a^2}{b^3}+\frac{b^2}{c^3}+\frac{c^2}{a^3}\geq 3$$

ML, t=259844, 2009

AI3 - 393

Let a, b, c be positive real numbers such that $a+b+c=3$. Prove that

$$\frac{a}{b^3+1}+\frac{b}{c^3+1}+\frac{c}{a^3+1}\leq\frac{3}{2}$$

Bin Zhao, ML, 2006

AI3 - 394

Let a, b, c be positive real numbers such that $a^2+b^2+c^2=3$. Prove that

$$\frac{a^2}{b^3+1}+\frac{b^2}{c^3+1}+\frac{c^2}{a^3+1}\geq\frac{3}{2}$$

ML, t=259844, 2009

------------------------------ **AI3 - 401** ------------------------------

Let a, b, c be positive real numbers such that $a+b+c=3$. Prove that

$$\frac{a}{b^3+2}+\frac{b}{c^3+2}+\frac{c}{a^3+2}\geq\frac{1}{6}(5+abc)$$

MathLinks, t=264216, 2009

------------------------------ **AI3 - 402** ------------------------------

For positive real numbers x, y, z holds $\frac{1}{x^2+1}+\frac{1}{y^2+1}+\frac{1}{z^2+1}=\frac{1}{2}$. Prove the inequality:

$$\frac{1}{x^3+2}+\frac{1}{y^3+2}+\frac{1}{z^3+2}<\frac{1}{3}$$

Serbia, Junior Balkan Team Selection Test, 2009

------------------------------ **AI3 - 403** ------------------------------

Let $x,y,z\in R^+$ and $x+y+z=3$. Prove that:

$$\frac{x^3}{y^3+8}+\frac{y^3}{z^3+8}+\frac{z^3}{x^3+8}\geq\frac{1}{9}+\frac{2}{27}(xy+yz+zx)$$

Iran, National Math Olympiad, 2008

------------------------------ **AI3 - 404** ------------------------------

Let a, b, c be positive real numbers such that $a+b+c\leq 1$. Prove that

$$\frac{a}{a^3+a^2+1}+\frac{b}{b^3+b^2+1}+\frac{c}{c^3+c^2+1}\leq\frac{27}{31}$$

Cao Minh Quang, CRUX 35(2) n. 3421, 2009

------------------------------ **AI3 - 405** ------------------------------

Let a, b, c be non-negative real numbers such that $a+b+c=2$ and $k\geq 1$. Prove that

$$\frac{\sqrt[k]{a}}{b^3+c^3}+\frac{\sqrt[k]{b}}{c^3+a^3}+\frac{\sqrt[k]{c}}{a^3+b^3}\geq 2$$

Tran Quoc Anh, ML, t=292594, 2009

AI3 - 411

If $a \ge b \ge c > 0$, then

$$\frac{a^3b}{a^3+b^3}+\frac{b^3c}{b^3+c^3}+\frac{c^3a}{c^3+a^3} \ge \frac{ab^3}{a^3+b^3}+\frac{bc^3}{b^3+c^3}+\frac{ca^3}{c^3+a^3}$$

AI3 - 412

Let a, b, c be positive real numbers such that $a+b+c=1$. Prove that

$$\frac{a^5+a^4+b^5}{a^3+b^3}+\frac{b^5+b^4+c^5}{b^3+c^3}+\frac{c^5+c^4+a^5}{c^3+a^3} \ge \frac{5}{6}$$

Panagiote Ligouras, ML, t=318005, 2009

AI3 - 413

Let a, b, c be positive real numbers such that $a+b+c=1$. Prove that

$$\frac{3a^5+2a^4+3b^5}{a^3+b^3}+\frac{3b^5+2b^4+3c^5}{b^3+c^3}+\frac{3c^5+2c^4+3a^5}{c^3+a^3} \ge 2$$

Panagiote Ligouras, MathLinks, t=250356, 2009

AI3 - 414

Let a, b, c be positive real numbers. Prove that

$$\frac{3a^3+abc}{b^3+c^3}+\frac{3b^3+abc}{c^3+a^3}+\frac{3c^3+abc}{a^3+b^3} \ge 6$$

Tran Quoc Anh, ML, t=262924, 2009

AI3 - 415

Let a, b, c be positive real numbers such that $a+b+c=1$. Prove that

$$\frac{b^5+3a^3+c^5+abc}{b^3+c^3}+\frac{a^5+3b^3+c^5+abc}{c^3+a^3}+\frac{b^5+3c^3+a^5+abc}{a^3+b^3} \ge \frac{19}{3}$$

Panagiote Ligouras, ML, t=297732, 2009

AI3 - 416

Let a, b, c be positive real numbers such that $a+b+c=1$. Prove that

$$\frac{3b^5+3a^3+3c^5+abc}{b^3+c^3}+\frac{3a^5+3b^3+3c^5+abc}{c^3+a^3}+\frac{3b^5+3c^3+3a^5+abc}{a^3+b^3} \ge 7$$

Panagiote Ligouras, MathLinks, t= 315857, 2009

AI3 - 423

If $a,b,c>0$ with $a+b+c=1$, prove that

$$\frac{a^7+b^7}{a^5+b^5}+\frac{b^7+c^7}{b^5+c^5}+\frac{c^7+a^7}{c^5+a^5}\geq\frac{1}{3}$$

AI3 - 424

Let $a,b,c>0$ and $abc=1$

$$\frac{ab}{a^5+b^5+ab}+\frac{bc}{b^5+c^5+bc}+\frac{ca}{c^5+a^5+ca}\leq 1$$

Unused problem in IMO 1996, EXC 2(4)+2(5), 1996

AI3 - 425

Let *a*, *b*, *c* be positive real numbers such that $a+b+c=1$. Prove that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq 9$$

AI3 - 426

Let *a*, *b*, *c* be positive real numbers. Prove that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq 2\left(\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}\right)$$

AI3 - 427

Let *a*, *b*, *c* be positive real numbers. Prove that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq 4\left(\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}\right)-\frac{9}{a+b+c}$$

AI3 - 428

If $a,b,c>0$, prove that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\leq\frac{a^8+b^8+c^8}{a^3b^3c^3}$$

G. C. Giri, CRUX n. 413

------------------------------- **AI3 - 435** -------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\ge 6\left(\frac{a}{3a^2+2b^2+c^2}+\frac{b}{3b^2+2c^2+a^2}+\frac{c}{3c^2+2a^2+b^2}\right)$$

------------------------------- **AI3 - 436** -------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\ge 6\left(\frac{1}{a^3+b^3+4}+\frac{1}{b^3+c^3+4}+\frac{1}{c^3+a^3+4}\right)$$

------------------------------- **AI3 - 437** -------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{1}{a}+\frac{4}{b}+\frac{9}{c}\ge\frac{36}{a+b+c}$$

------------------------------- **AI3 - 438** -------------------------------

Let a, b, c be positive real numbers such that $12\ge 21ab+2bc+8ca$. Then

$$\frac{1}{a}+\frac{2}{b}+\frac{3}{c}\ge\frac{15}{2}$$

Tran Nam Dung, Vietnam TST, 2001

------------------------------- **AI3 - 439** -------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{b}+\frac{a}{c}\ge 4\frac{a}{b+c}$$

------------------------------- **AI3 - 440** -------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{b}+\frac{b}{a}\ge\frac{a+c}{b+c}+\frac{b+c}{a+c}$$

AI3 - 447

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq\frac{a+b}{b+c}+\frac{b+c}{a+b}+1$$

AI3 - 448

Let a, b, c be positive real numbers such that $c\geq b\geq a$. Prove that

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq\frac{b}{a}+\frac{c}{b}+\frac{a}{c}$$

AI3 - 449

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq\frac{a+b}{a+c}+\frac{b+c}{b+a}+\frac{c+a}{c+b}$$

Indian-TST to the IMO 2002

AI3 - 450

Let a, b, c, k be positive real numbers. Prove that

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq\frac{a+kb}{a+kc}+\frac{b+kc}{b+ka}+\frac{c+ka}{c+kb}$$

Nguyen Viet Anh

AI3 - 451

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}+\frac{3}{2}$$

MathLinks, t=262433, 2009

AI3 - 452

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq\frac{1}{2}\left(\frac{b}{a+c}+\frac{b+2a}{b+c}+\frac{b+2c}{a+b}+\frac{5}{2}\right)$$

Panagiote Ligouras, MathLinks, t=302100, 2009

------------------------------- **AI3 - 459** -------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq 2\cdot\sqrt{\frac{a^2+b^2+c^2}{ab+bc+ca}+\frac{5}{4}}$$

Panagiote Ligouras, MathLinks, t=315855, 2009

------------------------------- **AI3 - 460** -------------------------------

Let a, b, c be positive real numbers such that $abc\leq 1$. Prove that

$$\frac{a}{c}+\frac{b}{a}+\frac{c}{b}\geq a+b+c$$

EXCALIBUR, 5(4), 2000

------------------------------- **AI3 - 461** -------------------------------

Let a, b, c be positive real numbers such that $abc=1$. Prove that

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq (a+1)(b+1)(c+1)-5$$

Aaron Pixton

------------------------------- **AI3 - 462** -------------------------------

Let a, b, c be positive real numbers such that $abc=1$. Prove that

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq \frac{1}{2}\left[(a+b+c)+\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\right]$$

------------------------------- **AI3 - 463** -------------------------------

Let a, b, c be positive real numbers. Prove that

$$\left(\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\right)^2\geq (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Great Britain Olympiad, 2005

------------------------------- **AI3 - 464** -------------------------------

Let a, b, c be positive real numbers. Prove that

$$\left(\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\right)^2\geq \frac{3}{2}\left(\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b}\right)$$

Romany-JTST, 2006

AI3 - 471

Let a, b, c be positive real numbers, show that

$$\frac{ab}{c}+\frac{bc}{a}+\frac{ca}{b}>2\cdot\sqrt[3]{a^3+b^3+c^3}$$

China-TST, 2008

AI3 - 472

Let a, b, c be positive real numbers such that $a^2+b^2+c^2=3$. Prove that

$$\frac{ab}{c}+\frac{bc}{a}+\frac{ca}{b}>1+\frac{a+b+c}{3}+\frac{2}{3}\cdot\sqrt[3]{a^3+b^3+c^3}$$

Panagiote Ligouras, MathLinks, t=310241, 2009

AI3 - 473

Let a, b, c be positive real numbers such that $a+b+c=8$. Prove that

$$\frac{ab}{c}+\frac{bc}{a}+\frac{ca}{b}>4\cdot\sqrt[6]{a^3+b^3+c^3}$$

Panagiote Ligouras, ML, t=311790, 2009

AI3 - 474

If $a, b, c>0$ and $a+b+c+\sqrt{abc}=4$, then

$$\sqrt{\frac{ab}{c}}+\sqrt{\frac{bc}{a}}+\sqrt{\frac{ca}{b}}>a+b+c$$

China-TST, 2007

AI3 - 475

Let a, b, c be positive real numbers. Prove that

$$\frac{a^2}{b}+\frac{b^2}{c}+\frac{c^2}{a}\geq a+b+c+\frac{4(a-b)^2}{a+b+c}$$

BMO, 2005

AI3 - 476

Let a, b, c be positive real numbers. Prove that

$$\frac{a^2}{b}+\frac{b^2}{c}+\frac{c^2}{a}\geq\frac{(a+b+c)\left(a^2+b^2+c^2\right)}{ab+bc+ca}$$

MathLinks, t=249523, 2009

-------------------------------- **AI3 - 483** --------------------------------

Let $a,b,c>0$ such that $a+b+c=1$. Prove:

$$\frac{a^2}{b}+\frac{b^2}{c}+\frac{c^2}{a}\geq 3\left(a^2+b^2+c^2\right)$$

Croatia TST 2007

-------------------------------- **AI3 - 484** --------------------------------

Given positive real numbers *a*, *b*, *c*, prove that

$$\frac{a^2}{b}+\frac{b^2}{c}+\frac{c^2}{a}\geq \frac{37\left(a^2+b^2+c^2\right)-19\left(ab+bc+ca\right)}{6\left(a+b+c\right)}$$

Michael Rozenberg, ML, t=296853, 2009

-------------------------------- **AI3 - 485** --------------------------------

Let *a*, *b*, *c* be positive real numbers. Prove that

$$\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}\geq \frac{a}{b}+\frac{b}{c}+\frac{c}{a}$$

1st Nordic Mathematical Olympiad

-------------------------------- **AI3 - 486** --------------------------------

Given positive real numbers *a*, *b*, *c*, prove that

$$\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}\geq 12-\frac{9\left(ab+bc+ca\right)}{a^2+b^2+c^2}$$

-------------------------------- **AI3 - 487** --------------------------------

Let *a*, *b*, *c* be positive real numbers. Prove that

$$\left(\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}\right)+\left(\frac{b^2}{a^2}+\frac{c^2}{b^2}+\frac{a^2}{c^2}\right)\geq \frac{1}{2}\left(\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\right)+\frac{1}{2}\left(\frac{b}{a}+\frac{c}{b}+\frac{a}{c}\right)+3$$

-------------------------------- **AI3 - 488** --------------------------------

Let *a*, *b*, *c* be positive real numbers. Prove that

$$\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}\geq \frac{2}{3}\left(a+b+c\right)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-\sqrt{\left(a^2+b^2+c^2\right)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)}$$

Panagiote Ligouras, MathLinks, t=310240, 2009

------------------------------- **AI3 - 495** -------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{a^3}{b^3}+\frac{b^3}{c^3}+\frac{c^3}{a^3}\geq\frac{a}{b}+\frac{b}{c}+\frac{c}{a},\quad a,b,c>0$$

------------------------------- **AI3 - 496** -------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{(-a+b+c)^3}{a}+\frac{(a-b+c)^3}{b}+\frac{(a+b-c)^3}{c}\geq a^2+b^2+c^2$$

------------------------------- **AI3 - 497** -------------------------------

Suppose that a, b, c are positive real numbers and $a^5+b^5+c^5=3$. Then

$$\frac{a^4}{b^3}+\frac{b^4}{c^3}+\frac{c^4}{a^3}\geq 3$$

------------------------------- **AI3 - 498** -------------------------------

Prove that for all positive real numbers a, b, c

$$\frac{a^5}{b}+\frac{b^5}{c}+\frac{c^5}{a}\geq 3\cdot\sqrt[3]{\left(\frac{a^6+b^6+c^6}{3}\right)^2}$$

Nguyen Thuc Vu Hoang, MathLinks, t=298305, 2009

------------------------------- **AI3 - 499** -------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{b}+\sqrt{\frac{b}{c}}+\sqrt[3]{\frac{c}{a}}\geq\frac{3}{2}$$

------------------------------- **AI3 - 500** -------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{a}{b}+2\cdot\sqrt{\frac{b}{c}}+3\cdot\sqrt[3]{\frac{c}{a}}\geq 6$$

------------------------------ **AI3 - 507** ------------------------------

Prove that

$$\sqrt{\frac{a+b}{c}}+\sqrt{\frac{b+c}{a}}+\sqrt{\frac{c+a}{b}}\geq 3\sqrt{2}$$

where a, b, c are positive real numbers.

Šefket Arslanagić, Die WURZEL, n. η50

------------------------------ **AI3 - 508** ------------------------------

Let a, b, c be positive real numbers. Prove that

$$\sqrt{\frac{a+b}{c}}+\sqrt{\frac{b+c}{a}}+\sqrt{\frac{c+a}{b}}\geq \sqrt{6\cdot\frac{a+b+c}{\sqrt[3]{abc}}}$$

Pham Huu Duc, Mathemtical Reflections n. S41, 2007

------------------------------ **AI3 - 509** ------------------------------

Let a, b, c be positive real numbers. Prove that

$$\sqrt{\frac{a+b}{c}}+\sqrt{\frac{b+c}{a}}+\sqrt{\frac{c+a}{b}}\geq \sqrt{\frac{16(a+b+c)^3}{3(a+b)(b+c)(c+a)}}$$

Vo Quoc Ba Can, Mathemtical Reflections n. O43, 2007

------------------------------ **AI3 - 510** ------------------------------

Let a, b, c be positive real numbers. Prove that

$$\sqrt{\frac{a+b}{c}}+\sqrt{\frac{b+c}{a}}+\sqrt{\frac{c+a}{b}}\geq 2(a+b+c)\cdot\sqrt[4]{\frac{2}{\sqrt[3]{abc}\,(a+b)(b+c)(c+a)}}$$

Panagiote Ligouras, MathLinks, t=252271, 2009

------------------------------ **AI3 - 511** ------------------------------

Let a, b, c be positive real numbers. Prove that

$$\sqrt{\frac{a+b}{c}}+\sqrt{\frac{b+c}{a}}+\sqrt{\frac{c+a}{b}}\geq 2\cdot\sqrt[4]{\frac{6(a+b+c)^3}{(a+b)(b+c)(c+a)}}$$

Panagiote Ligouras, MathLinks, t=308751, 2009

------------------------------ **AI3 - 512** ------------------------------

Let a, b, c be positive real numbers. Prove that

$$\sqrt{\frac{a+b}{c}}+\sqrt{\frac{b+c}{a}}+\sqrt{\frac{c+a}{b}}\geq 2\cdot\sqrt[6]{\frac{9(a+b+c)^4}{\sqrt[3]{abc}\,(a+b)(b+c)(c+a)}}$$

Panagiote Ligouras, MathLinks, t=297725, 2009

AI3 - 519

Let a, b, c be positive real numbers, prove that

$$\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b}-4\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)\geq 1-\frac{8abc}{(a+b)(b+c)(c+a)}$$

Cezar Lupu & Cosmin Pohoata, RSME, 11(1), n. 97, 2008

AI3 - 520

For a, b, $c > 0$, if $abc = 1$, then show that

$$\frac{b+c}{\sqrt{a}}+\frac{c+a}{\sqrt{b}}+\frac{a+b}{\sqrt{c}}\geq \sqrt{a}+\sqrt{b}+\sqrt{c}+3$$

EXCALIBUR, 5(3) & 5(4), 2000

AI3 - 521

Let a, b, c be positive real numbers, prove that

$$\frac{(a+b)^3}{c}+\frac{(b+c)^3}{a}+\frac{(c+a)^3}{b}\geq 8(ab+bc+ca)$$

AI3 - 522

Let a, b, c be positive real numbers, prove that

$$\frac{(a+b)^3}{c}+\frac{(b+c)^3}{a}+\frac{(c+a)^3}{b}\geq 8\left(a^2+b^2+c^2\right)$$

Romania, order 8, 2003

AI3 - 523

Let a, b, c be positive real numbers, prove that

$$\frac{(a+b)^3}{c}+\frac{(b+c)^3}{a}+\frac{(c+a)^3}{b}\geq 2\left(a^2+b^2+c^2\right)+2(a+b+c)^2$$

Panagiote Ligouras, MathLinks, t=297731, 2009

AI3 - 524

Let a, b, c be positive real numbers such that $abc=1$, prove that

$$\frac{a^2+b^2}{c}+\frac{b^2+c^2}{a}+\frac{c^2+a^2}{b}\geq 6$$

Gazeta Matematică, 2002

-------------------------------- **AI3 - 531** --------------------------------

If a, b, c are positive numbers, then

$$a+b+c+\frac{a^2}{b}+\frac{b^2}{c}+\frac{c^2}{a}\ge\frac{6\left(a^2+b^2+c^2\right)}{a+b+c}$$

Pham Huu Duc, MathLinks, 2006

-------------------------------- **AI3 - 532** --------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{a^3b+ab^3}{c}+\frac{b^3c+bc^3}{a}+\frac{c^3a+ca^3}{b}\ge 6abc$$

-------------------------------- **AI3 - 533** --------------------------------

Let a, b, c be positive real numbers. Prove that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\ge 2\left(\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}\right)$$

-------------------------------- **AI3 - 534** --------------------------------

Let a, b, c be positive real numbers such that $a\ge b\ge c$. Prove that

$$\frac{a^2b}{c}+\frac{b^2c}{a}+\frac{c^2a}{b}\ge a^2+b^2+c^2$$

Vietnam, 29ts MO, 1991

-------------------------------- **AI3 - 535** --------------------------------

Let a, b, c be positive real numbers such that $a+b+c=6$. Prove that

$$\left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{c}\right)^2+\left(c+\frac{1}{a}\right)^2\ge\frac{75}{4}$$

ARML, 1987

-------------------------------- **AI3 - 536** --------------------------------

Let a, b, c be positive real numbers such that $ab+bc+ca=1$. Then

$$\left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{c}\right)^2+\left(c+\frac{1}{a}\right)^2\ge 16$$

Mircea Becheanu, Mathematical Reflections, n. J57, 2007

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{4}{3}(a+b+c)\geq 7$$

------------------------------ **AI3 - 543** ------------------------------

Let a, b, c be positive real numbers such that $a+b+c=1$. Prove that

$$\frac{a+b^2}{c}+\frac{b+c^2}{a}+\frac{c+a^2}{b}\geq 4$$

------------------------------ **AI3 - 544** ------------------------------

Let a, b, c be positive real numbers. Prove that

$$\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right)\geq\left(a+\frac{1}{b}\right)\left(b+\frac{1}{c}\right)\left(c+\frac{1}{a}\right)$$

------------------------------ **AI3 - 545** ------------------------------

Let a, b, c be positive real numbers such that $a+b+c=1$. Prove that

$$\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right)\geq 64$$

Yugoslavia, 30th MO, 1989

------------------------------ **AI3 - 546** ------------------------------

Let a, b, c be arbitrary positive real numbers. Prove that

$$\left(1+\frac{a}{b}\right)\left(1+\frac{b}{c}\right)\left(1+\frac{c}{a}\right)\geq 2+\frac{2(a+b+c)}{\sqrt[3]{abc}}$$

APMO, 1998

------------------------------ **AI3 - 547** ------------------------------

Let a, b, c be arbitrary positive real numbers. Prove that

$$\left(1+\frac{a^2}{b}\right)\left(1+\frac{b^2}{c}\right)\left(1+\frac{c^2}{a}\right)\geq(1+a)(1+b)(1+c)$$

APMO

------------------------------ **AI3 - 548** ------------------------------

Let a, b, c be arbitrary positive real numbers. Prove that

$$a+b+c+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\le 3+\frac{a}{b}+\frac{b}{c}+\frac{c}{a}$$

Todor Mitev, CRUX, n. 3236

------------------------------ **AI3 - 555** ------------------------------

Let a, b, c be positive real numbers such that $abc=1$. Prove that

$$a^2+b^2+c^2+6\ge \frac{3}{2}\left(a+b+c+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Vasile Cirtoaje, ML, 2006

------------------------------ **AI3 - 556** ------------------------------

Let a, b, c be positive real numbers such that $abc=1$. Prove that

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\ge (a+b+c)+\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-3$$

EXCALIBUR, 12(4), 2007

------------------------------ **AI3 - 557** ------------------------------

Let a, b, c be positive real numbers such that $abc=1$. Prove that

$$a^2+b^2+c^2+\frac{a}{b}+\frac{b}{c}+\frac{c}{a}+9\ge 2\left(a+b+c+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Panagiote Ligouras, MathLinks, t=252656, 2009

------------------------------ **AI3 - 558** ------------------------------

Let a, b, c be positive real numbers such that $abc=1$. Prove that

$$a+b+c+\frac{2}{3}\left(\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\right)\ge 2+\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

------------------------------ **AI3 - 559** ------------------------------

Let a, b, c be positive real numbers. Prove that

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\ge 9$$

------------------------------ **AI3 - 560** ------------------------------

Let a, b, c be positive real numbers. Prove that

If $y \ge a,b,c \ge x > 0$, then

$$\frac{(2x+y)(x+2y)}{xy} \ge (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \ge 9$$